Analytical Model Improvement Using Singular Value Decomposition With a One-Dimensional Line Searching Technique

Matthew F. Orr, Jr./ED23
205–544–1534
E-mail: matthew.orr@msfc.nasa.gov

The goal of analytical model improvement is to systematically improve a finite element model so that predicted responses such as mode shapes and natural frequencies closely match those measured by actual experimental tests. The updated finite element model can then be used to more accurately predict transient response loads and displacements. In general, both the experimental data and the analytical model may contain errors, and the improvement process should detect and minimize these differences.

The software code AMI (analytical model improvement) used in this study, was developed for both structural optimization applications and for analytical model improvement applications. AMI consists of a UNIX script file which calls NASTRAN to provide structural data and also calls several FORTRAN codes that calculate improved or optimized design parameters.

The methodology developed for AMI uses several advanced mathematical techniques. A search direction is first calculated with a linear least-squares solution, using singular value decomposition (SVD). Second order information is then utilized, in a one-dimensional line search, to determine a step size which yields the optimal match between experimental and analytical data. Two approximation functions are considered for use in the one-dimensional line search.

The analytical model improvement analysis, in its simplest form, is essentially a linear least-squares problem. That is because the number of Eigenvector degrees-of-freedom and Eigenvalues, measured during the modal test, usually greatly exceed the number of design variables that are to be solved for. In matrix notation, this can be expressed by a linear equation as,

$$\Delta = J\,\delta$$

where Δ is a vector containing the differences between experimental measured and analytical quantities such as eigenvalues and eigenvectors. The vector δ contains the design parameter changes to be solved for in order that the experimental and analytical quantities are in agreement. The Jacobian matrix J contains the sensitivities of the analytical modal quantities with respect to the design parameters.

Using SVD the Jacobian matrix J can be written as the product of three matrices,

$$J = U\,W\,V^T$$

And the above linear equation can be solved as,

$$\delta = V\,W^{-1}\,U^T\,\Delta$$

The solution that this SVD method yields, is the so-called minimal p-2 norm residual error solution. It basically represents the best fit of the analytical model to the experimental data possible. Any ill conditioning present in the system of equations becomes readily apparent when examining the singular values or the resulting condition number which is the ratio of the largest singular value divided by the smallest singular value. Singular values less than a given tolerance level are discarded, thus removing any ill conditioning, and insuring the accuracy of the solution.

In practice, the relationship between experimental and analytical eigenquantities is often not well approximated by the above linear equation. Significant nonlinearities may exist. The problem then becomes one of nonlinear data fitting which can generally only be solved by iteration. For this reason, the analytical model improvement problem is best approximated as a series of sequential linear least-squares problems. With this method, the FEM is updated during each analytical model improvement cycle, and if convergence is not achieved, the process of calculating sensitivities and solving the resulting linear least-squares equations is repeated.

Sometimes the linear least-squares incremental solution, δ will produce a more accurate parametric model than existed originally. Often times, however it may be wildly divergent. It can be shown that there exists a step-size parameter α, such that α times the vector δ will yield an incremental solution for the design variables,

$$\delta_{\text{incremental}} = \alpha\,\delta$$

with a residual error that is guaranteed to be less than the previous residual. If the residual error can be sequentially minimized, the final resulting solution should be the best solution possible.

In this mathematical scheme, the linear solution δ is used as the search direction and the step size parameter α represents what portion of the linear solution is to be used. The trick then, is to determine what value of the step-size parameter α to use. With a constant search direction δ, the estimated design variable values are solely a function of the parameter α,

$$\delta(\alpha)_{\text{estimated}} = \alpha\,\delta + \delta_{\text{initial}}$$

Approximations for Eigenvalues and Eigenvector degrees-of-freedom, which are a function of the estimated design variables, $\delta(\alpha)_{\text{estimated}}$ are also required. Two methods for the approximations were evaluated. The first method uses a special version of the method of moving asymptotes (MMA) to approximate the eigenquantities. The second method uses a generalized quadratic equation (GQE) for the approximations. The approximate eigenquantity functions are such that they exactly match the eigenquantities and their first and second derivatives at the initial design parameters.

Using the approximation functions (MMA or GQE), the p-2 norm residual error between the estimated and the test measured eigenquantities is next determined. This estimated p-2 norm residual error is now only a function of the parameter α.

A one-dimensional line search optimization routine in the International Math and Statistics Library (IMSL) is next used to find the value of α, which minimizes the estimated residual error. This value of the parameter, say α^*, is then used to get an improved estimate of the design variables.

Both approximation functions, MMA and GQE, use second order eigenquantity information, namely primary second derivatives, determined through a forward finite difference technique. Whenever a second derivative term is zero or is negative, a small positive number is substituted. This provides for a so-called convex approximation which provides for a unique solution.

A fixed-base modal survey was conducted at Marshall Space Flight Center on December 11, 1991, which experimentally determined the lower-frequency dynamic characteristics of the Small Expendable Deployer System (SEDS). Figure 49 shows the finite element model for the SEDS test article including several of the major structural components. This analytical finite element model contains approximately 6,600 degrees-of-freedom. The first two natural frequencies and mode shapes were used for this correlation study; the corresponding frequency range of interest was 50 Hz and less. The first test and analysis mode shape was a pitch mode of the canister about the x-axis. The second was a lateral canister translation in the x direction.

Table 3 shows the design variables that were chosen to be updated in this study. Design variable number one is the canister delta mass moment of inertia about the x- or z-axis. This variable augments the finite element model prediction of the mass moment of inertia of the tether which is inside of the canister. It tends to compensate for modeling the tether with using only two lumped masses connected by very stiff bar

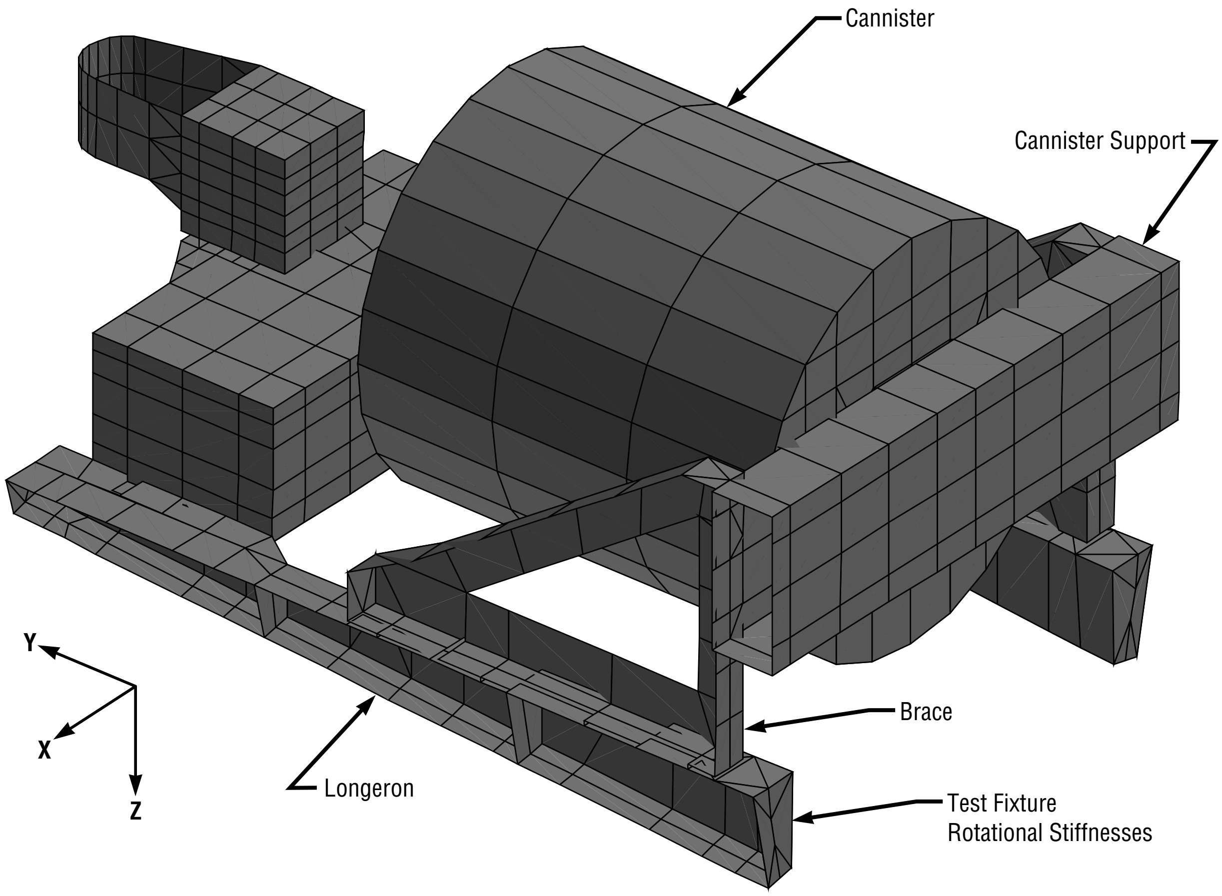

Figure 49.—Small Expendable Deployer System (SEDS) finite element model

Table 3.—*Design parameter definitions.*

Design Parameter	Description
1	Cannister Delta Moment of Inertia
2	Aft Test Fixture Rotational Stiffness $K_{\theta y}$
3	Brace to Torquebox to Cannister Stiffness K
4	Aft Test Fixture Rotational Stiffness $K_{\theta x}$
5	Longeron Stiffness E (modulus of elasticity)
6	Aft Test Fixture Rotational Stiffness $K_{\theta z}$
7	Brace Stiffness E (modulus of elasticity)
8	Cannister Support Stiffness E (modulus of elasticity)

elements; the mass moments of the tether's inertia had not been determined exactly. The modulus of elasticity of the longeron, of the canister support or torque tube, and of the brace were also chosen as design variables. Three rotational stiffnesses design variables, at the aft ends of the longerons, represent test fixture effects. Another design parameter is a bearing stiffness between the brace and canister support and canister.

With the eight design parameters chosen, the unbounded linear solution was prone to fairly large excursions. This is partly due to combining mass or inertia design variables with stiffness variables during the analysis. Using physically realistic design parameters is an important requirement for this type of analysis, however, if the results produced are to be meaningful. Figure 50 shows several analyses using four different options for selecting the step-length parameter α. A constant move-limit assumption indirectly specifies the parameter α since the largest design variable is limited to a given fraction

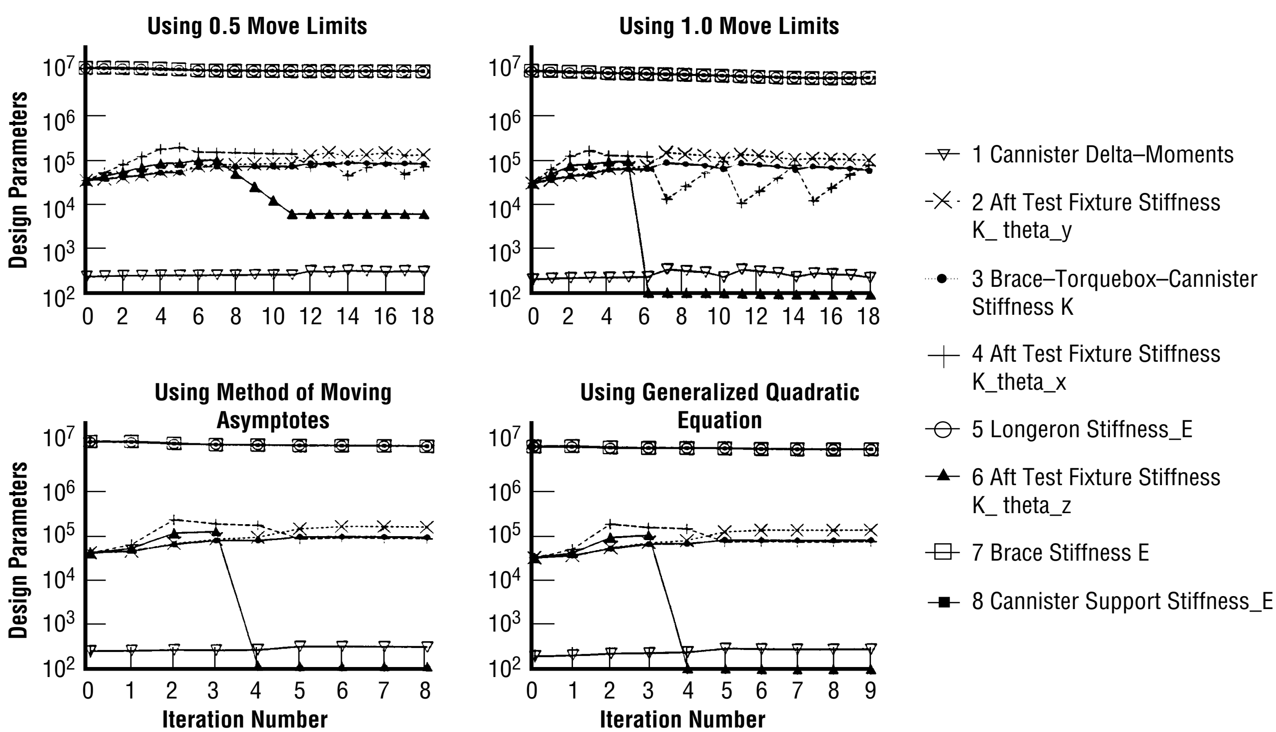

Figure 50.—Analytical model improvement analysis—design parameters versus iteration number for various methods of computing a.

of its previous value and other design parameters are ratioed accordingly. This move limit option is often used in structural optimization problems. The move-limit options using move-limits of 0.5 and of 1.0, however, did fail to converge even after 18 iterations.

The processes using the method of moving asymptotes and generalized quadratic equation converge in eight and nine iterations respectively. These results show the advantage of using approximation functions with second order information.

The AMI software also includes a FORTRAN code that monitors various test-related parameters such as modal assurance criteria numbers (MAC) and cross-orthogonality numbers. The final MAC numbers predicted were 0.994 and 0.993 for the first two modes. The diagonal cross orthogonality numbers between analysis and test were 0.999 for both sets of modes; the off-diagonal values were only 0.004 and 0.003. And frequency errors were minimized to 0.05 percent for both modes. Acceptable values for frequency errors are usually three to five percent, and diagonal cross-orthogonality numbers are usually required to be 0.900 or greater; off diagonals should be less than 0.1. The above results predicted by AMI are significantly better than these requirements.

The second order approximation functions were very effective for reaching a converged solution in a reasonable number of iterations. The accuracy of the solution was still somewhat in question, however. For this reason, six simulated test cases were designed where the target solution was known before the AMI analysis was begun. Table 4 gives results of this study. The MMA determination of the step size α was used for this study since it seemed to yield somewhat better results than the GQE method.

Table 4.—*SEDS simulated data test cases.*

Test Case Number	Test Case Description		Iterations Required to Converge	Maximum % Error in Predicted Design Variables
	Design Variable	Changes From Nominal Case		
1	1–	+100%	4	0.0
2	3–	+100%	4	0.0
3	5–	+13.3%	3	0.1
4	8–	+13.3%	3	0.1
5	1– 3– 5–	+100% +100% +13.3%	6	0.1
6	1– 2– 3– 4– 5– 6– 7– 8–	+100% +100% +100% +100% +100% +66.7% −44.4% −66.7%	7	0.2

For test case number one, the first design variable was increased by 100 percent from its nominal value, i.e., increased from 268.7 to 537.4 lb/in^2, as given in weight units. Modal data were calculated for this configuration and used as experimental test data. The analytical model improvement analysis matched the design parameters for this test case in four iterations with negligible errors.

For every one of the six test cases in fact, each of the design parameters was predicted to the correct value with negligible errors. The most difficult test case considered, number six, took more iterations (seven) to converge—as would be expected—since this configuration had been substantially modified from the nominal configuration.

The AMI software is a useful tool that can be used by the analyst for the sometimes difficult task of analytical model improvement. The approximation functions, using second order information, converge to an accurate solution in a reasonable number of iterations. Improved design variables calculated by AMI are both physically realizable and also very realisitic.

Orr, M.F., Jr.: “Analytical Model Updating Using Singular Value Decomposition With a One-Dimensional Line Searching Technique.” AIAA/ASME/AHS Adaptive Structures Forum, Salt Lake City, UT, 1996.

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Biographical Sketch: Matthew F. Orr, Jr. has worked in the aerospace engineering field as a structural dynamicist for approximately 23 years after serving a 3-year tour in the U.S. Army. He received a B.S. in aerospace engineering from University of Kansas in 1973 and received a master’s of mechanical engineering with a perfect straight-A average from the University of Utah in 1978. He has worked at Beech Aircraft, Thiokol, Martin Marietta, McDonald Douglas, and has been with NASA/MSFC for over 5 years. He recently was awarded the Professional Engineering License from the State of Alabama.